

# Factorization and Resummation for Exclusive Jet Cross Sections

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Work with Iain Stewart, Wouter Waalewijn,  
Carola Berger, Claudio Marcantonini

[arXiv:0910.0467, arXiv:1002.2213

arXiv:1004.2489, arXiv:1005.4060, arXiv:1006.xxxx]



# Outline

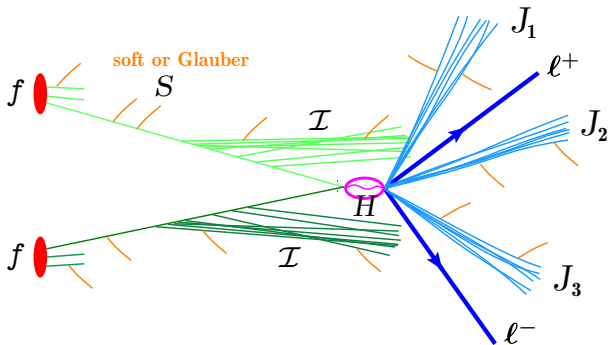
- 1 Jet Vetos and ISR
- 2 Beam Thrust and Beam Functions
- 3 NNLL Results for Drell-Yan and Higgs
- 4 Outlook

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# Hard Interaction at LHC

Higgs and New physics hide at short distances in hard interaction



Factorization: “Cross section can be computed by combining separate pieces”

$$d\sigma = \underbrace{\text{PDFs} \otimes \text{ISR}}_{\text{initial-state parton shower}} \otimes \text{hard interaction} \otimes \text{FSR} \otimes \text{soft radiation}$$

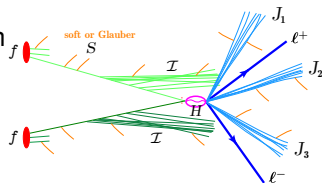
MC: initial-state parton shower      matrix-element generator      final-state parton shower      hadronization underlying event

# Why Veto Jets?

**Hard interaction** identified by looking for signal with characteristic number of **jets** plus **leptons/photons**

⇒ Want to measure *exclusive* jet cross section

$$pp \rightarrow X L + N \text{ jets}$$



Background discrimination often requires a *veto on additional jets*

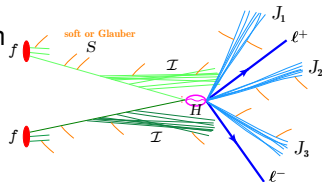
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  - ▶  $H \rightarrow WW$  dominated by  $t\bar{t} \rightarrow WWb\bar{b}$  by  $\sim 1 : 40$

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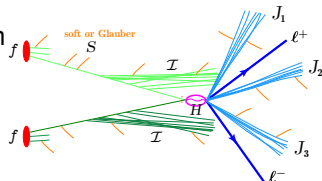
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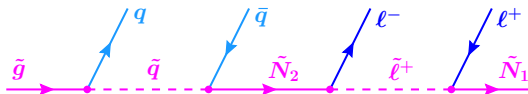
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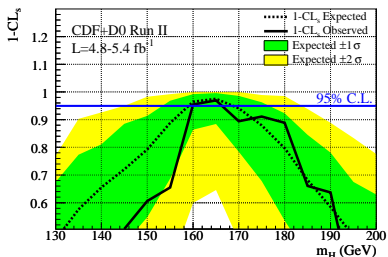


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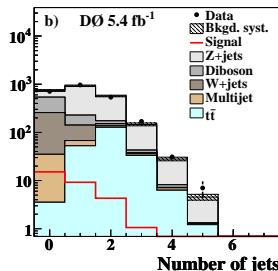
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  - ▶  $H \rightarrow \gamma\gamma$  has huge backgrounds from  $pp \rightarrow jj$ ,  $pp \rightarrow j\gamma$
- Reconstructing new-physics masses and decay chains
  - ▶ Additional jets cause large combinatorial backgrounds



# Example: $gg \rightarrow H \rightarrow WW$



[CDF+DØ (arXiv:1001.4162)]



[DØ (arXiv:1001.4481)]

Want  $gg \rightarrow H + 0 \text{ jets}$  to eliminate  $gg \rightarrow t\bar{t} \rightarrow WWb\bar{b}$

- Run jet algorithm and veto all events having jets with  $p_T^{\text{jet}} > p_T^{\text{cut}} \simeq 20 \text{ GeV}$  and  $|\eta^{\text{jet}}| < \eta^{\text{cut}} \simeq 2.5$

Tevatron excludes  $m_H \simeq 165 \text{ GeV}$  at 95% CL

- Dominant sensitivity from 0-jet sample

⇒ Setting limits requires reliable theory predictions and uncertainties  
(Theory uncertainties have been questioned [Baglio, Djouadi])

# Inclusive vs. Exclusive Jet Cross Section

## Inclusive jet cross section

- Can be obtained from fixed-order partonic calculation

$$\sigma^{\text{hadronic}} = \sum_{i,j} \sigma_{ij}^{\text{partonic}} \otimes f_i \otimes f_j$$

- Tree-level matrix element for  $ij \rightarrow N$  partons corresponds to LO inclusive (“total”) cross section to produce  $N$  or more jets

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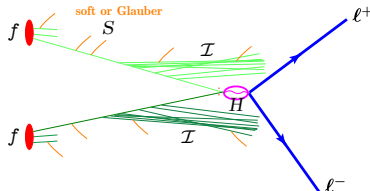
## Exclusive jet production *very different* from inclusive jet production

- Jet veto imposes strong phase space restriction, causing potentially large double logarithms  $\alpha_s^n \ln^{m \leq 2n}(p_T^{\text{cut}}/Q)$
- ⇒ Fixed-order perturbation theory may not be enough  
(One reason for attaching parton shower to a fixed-order calculation)
- ⇒ Theory uncertainties?
- ⇒ Relevant factorization theorem?

# Hadronic Initial-State Radiation

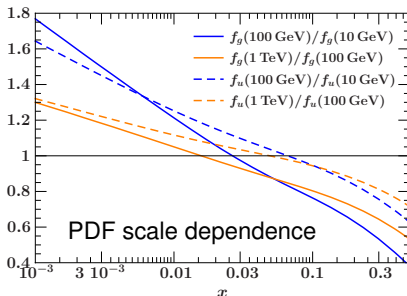
## Hadronic ISR has important effects

- Modifies parton luminosity available for **hard interaction**
  - Causes additional jets unrelated to hard interaction (incoming gluons radiate a lot)
- ⇒ Jet veto affects cross section by restricting ISR



PDF DGLAP evolution is insufficient to sum double logarithms from ISR

- Proper scale to evaluate PDFs?



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# Implementing Jet Vetos in Theory Predictions

## Jet veto via jet algorithm yields complicated phase-space cuts

- Currently rely on parton shower Monte Carlo to sum leading logs
  - ▶ ISR modelled by initial-state shower (less tested than final-state one)
  - ▶ MC@NLO, POWHEG: fixed NLO + parton-shower LL summation  
[Frixione, Webber; Nason *et al.*]
- FEHiP, HNNLO: NNLO parton-level Monte Carlos for  $gg \rightarrow H \rightarrow WW$   
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## Want to implement jet veto by cutting on simple kinematic variable

⇒ Analytic control of phase-space restriction from jet veto allows for

- 1 Systematic summation of phase-space logs  
(beyond parton shower and leading log)
- 2 Theory treatment of soft effects  
(beyond hadronization and underlying event models)
- 3 Better handle on theory uncertainties

# Isolated Drell-Yan: $pp \rightarrow X\ell^+\ell^- + 0 \text{ jets}$

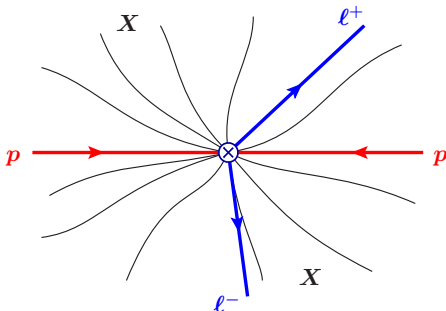
Hard interaction  $q\bar{q} \rightarrow \gamma/Z \rightarrow \ell^+\ell^-$

- Variables

$$Q^2 = (p_{\ell^+} + p_{\ell^-})^2$$

$Y = \text{dilepton rapidity}$

- Hard scale  $\mu_H \simeq Q = \sqrt{Q^2}$



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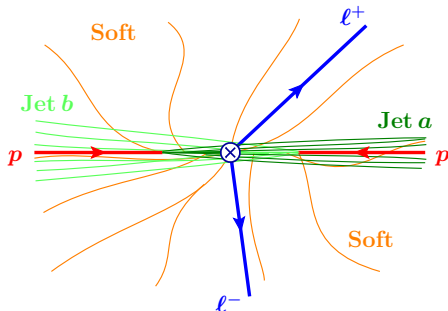
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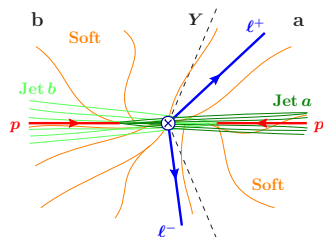
Impose veto on central jets:  $pp \rightarrow \gamma/Z + 0 \text{ jets}$

- Hadronic final state  $X$  dominated by ISR
- Energetic radiation only in forward direction (at measurable rapidities)
- Soft radiation everywhere (underlying event)

⇒ Simplest process to study jet veto and ISR

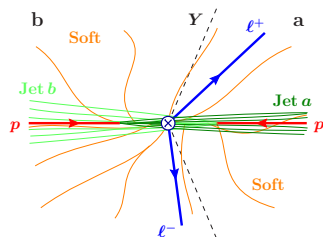
# Definition of Beam Thrust

$$\begin{aligned}\tau_B &= \frac{1}{Q} \sum_k |\vec{p}_{kT}| e^{-|\eta_k - Y|} \\ &= \frac{1}{Q} \left[ e^{-Y} \sum_{\eta_k < Y} (E_k + p_k^z) + e^Y \sum_{\eta_k > Y} (E_k - p_k^z) \right]\end{aligned}$$



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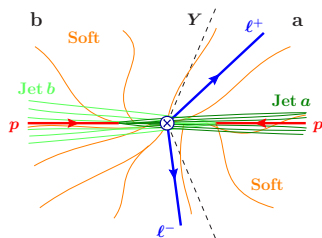
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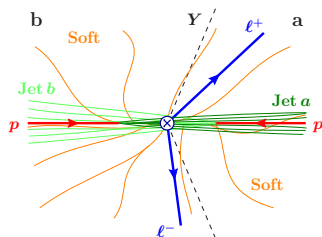
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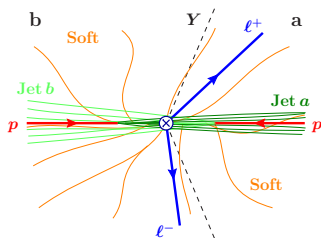
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Require  $\tau_B \ll 1$

- Allows **soft** & **forward energetic** but no **central energetic** radiation

⇒ Provides **central jet veto** via simple kinematic variable

# Beam Thrust Factorization Theorem

Factorization theorem for  $\tau_B \ll 1$

$$(x_{a,b} = (Q/E_{\text{cm}})e^{\pm Y})$$

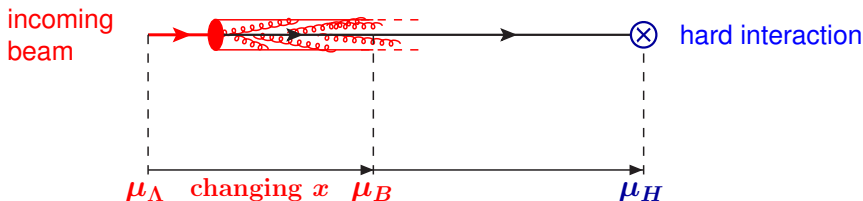
$$\begin{aligned} \frac{d\sigma}{dQ dY d\tau_B} &= \frac{8\pi\alpha_{\text{em}}^2}{9E_{\text{cm}}^2 Q} \sum_{ij=\{q\bar{q}, \bar{q}q\}} H_{ij}(Q^2, \mu) \\ &\times \int dt_a B_i(t_a, x_a, \mu) \int dt_b B_j(t_b, x_b, \mu) S_B^{q\bar{q}}\left(\tau_B - \frac{t_a + t_b}{Q^2}, \mu\right) \\ &\times [1 + \mathcal{O}(\Lambda_{\text{QCD}}/Q, \tau_B)] \end{aligned}$$

Function		describes	at scale
Hard	$H_{ij}(Q^2, \mu)$	hard virtual radiation	$\mu_H \simeq -iQ$
Beam	$B_i(t, x, \mu)$	virtual & real energetic ISR	$\mu_B \simeq \sqrt{\tau_B} Q$
Soft	$S_B(\tau_B^{\text{soft}}, \mu)$	virtual & real soft radiation	$\mu_S \simeq \tau_B Q$

- Large logs from central jet veto are  $\alpha_s^n (\ln^m \tau_B) / \tau_B$  or  $\alpha_s^n \ln^m \tau_B^{\text{cut}}$ 
  - Summed by RG evolution of  $H_{ij}(\mu)$ ,  $B_i(\mu)$ ,  $B_j(\mu)$ ,  $S_B(\mu)$

# Physical Picture of Initial State

Measurement probes PDFs at some intermediate scale  $\mu_B$

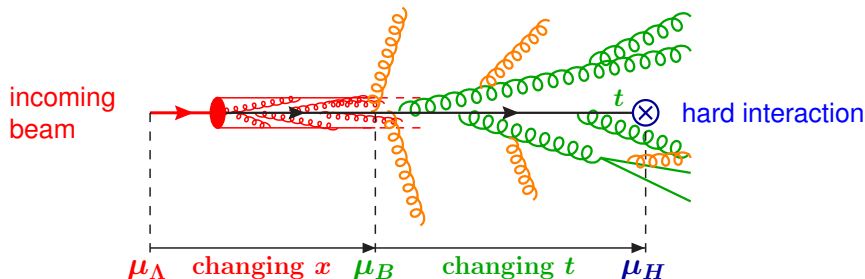


$\mu < \mu_B$ : On-shell partons “inside” incoming proton

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$\mu < \mu_B$ : On-shell partons “inside” incoming proton

- ISR captured by PDF evolution, redistributes momentum fraction  $x$

$\mu > \mu_B$ : Off-shell parton with virtuality  $-t < 0$  part of incoming jet

- Colliding parton emits **collinear** and **soft** ISR “outside” proton
- ISR goes into final state and is measured by jet veto

# Beam Function Calculation

$\mu = \mu_B$ : Can calculate  $B_i(\mu_B)$  perturbatively in terms of  $f_j(\mu_B)$

$$B_i(t, x, \mu_B) = \sum_j \int_x^1 \frac{d\xi}{\xi} \mathcal{I}_{ij}\left(t, \frac{x}{\xi}, \mu_B\right) f_j(\xi, \mu_B) = \delta(t) f_i(x, \mu_B) + \dots$$

- $\mathcal{I}_{ij}$  computed to NLO for all  $ij$

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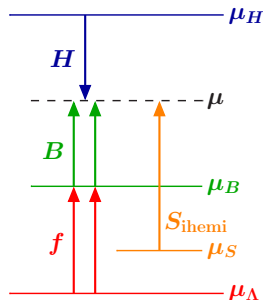
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$\mu > \mu_B$ : PDF evolution replaced by beam-function evolution

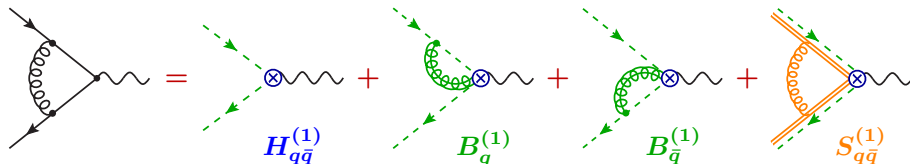
$$\mu \frac{d}{d\mu} B_i(t, x, \mu) = \int dt' \gamma_B^i(t - t', \mu) B_i(t', x, \mu)$$

$$\gamma_B^i(t, \mu) = -2\Gamma_{\text{cusp}}^i(\alpha_s) \frac{1}{\mu^2} \left[ \frac{\theta(t)}{t/\mu^2} \right]_+ + \gamma_B^i(\alpha_s) \delta(t)$$

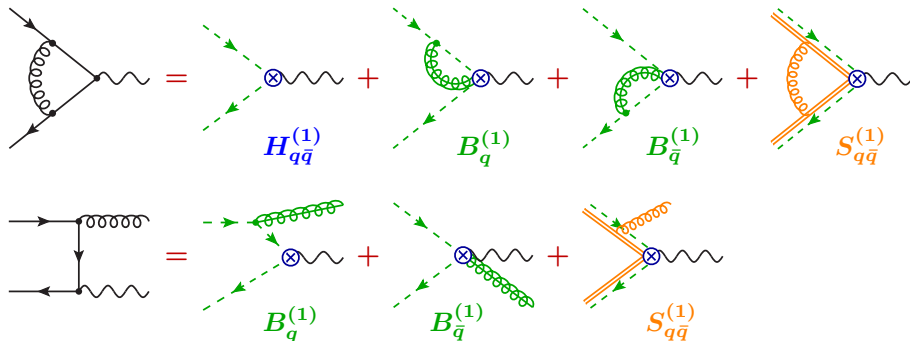
- Sums double logs of  $t$
- No mixing in  $x$  or  $i$  (in contrast to PDF)
- $\gamma_B^i(t) = \gamma_J^i(t)$ 
  - known to 3 loops [from Moch, Vermaseren, Vogt]



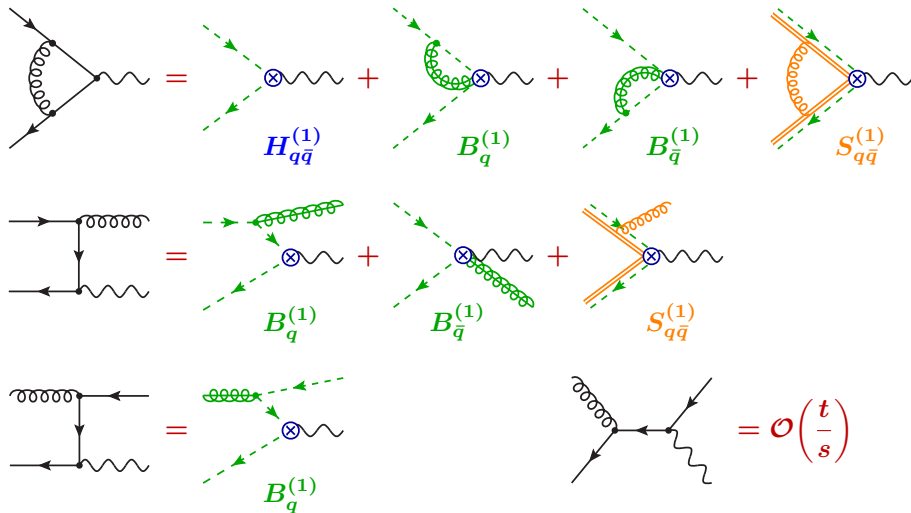
# Correspondence with Fixed-Order Calculation



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$\Rightarrow B(t)$  resums ISR  $t$ -channel singularities

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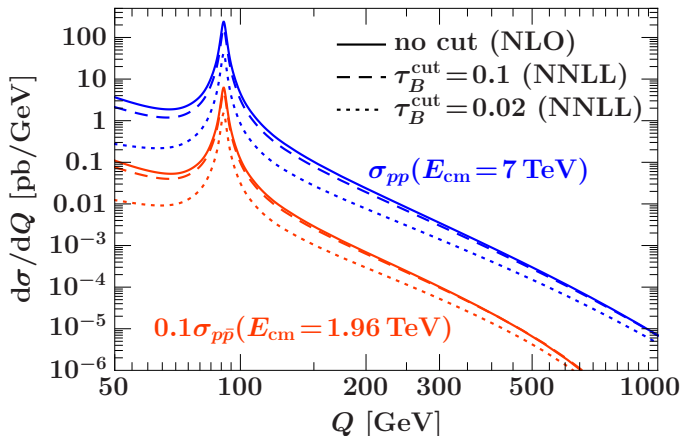
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# Drell-Yan Cross Section With Central Jet Veto

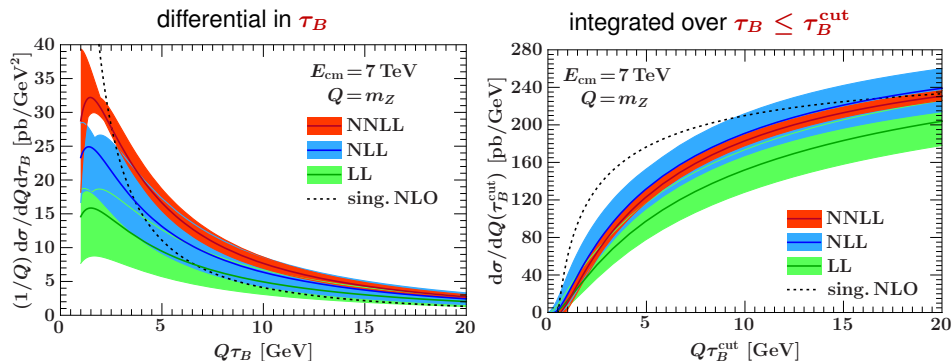
Compare cross section (integrated over  $Y$ )

[using MSTW2008 NLO PDFs]

- inclusive without cut at NLO
- with jet veto  $\tau_B^{\text{cut}} = \{0.1, 0.02\}$  at NNLL



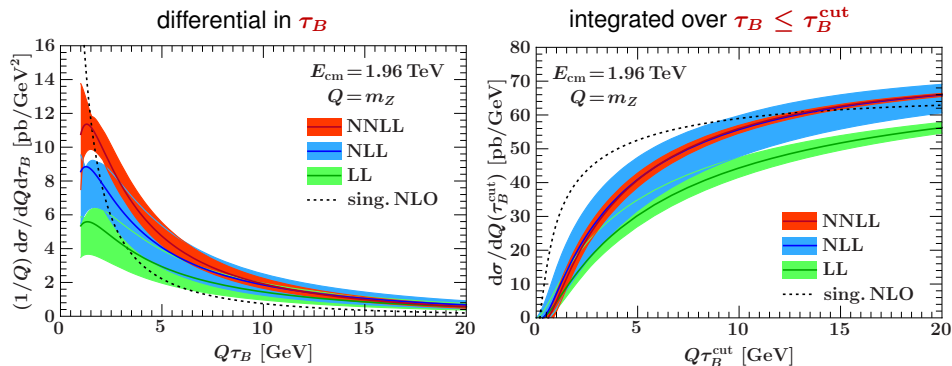
# Drell-Yan Beam Thrust Cross Section



Cross section dominated by small  $\tau_B$  where summing  $(\ln \tau_B)/\tau_B$  or  $\ln^2 \tau_B^{\text{cut}}$  is important

- Perturbative uncertainties are envelope of separate  $\mu_H$ ,  $\mu_B$ ,  $\mu_S$  variation
- **Soft function** is perturbative in tail, becomes nonperturbative below peak
- Good convergence, significantly improved by summing constant  $\pi^2$  terms in hard virtual corrections

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# Beam Thrust for Higgs Production

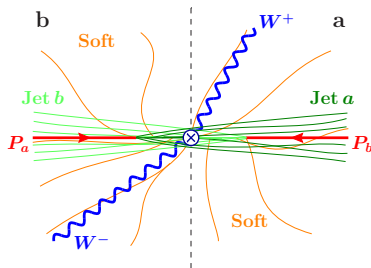
Higgs production:  $gg \rightarrow H + 0 \text{ jets}$

$$\mathcal{T}_B^{\text{cm}} = \sum_k |\vec{p}_{kT}| e^{-|\eta_k|} = \sum_k (E_k - |p_k^z|)$$

Factorization theorem for  $\mathcal{T}_B^{\text{cm}} \ll m_H$

$$x_{a,b} = (m_H/E_{\text{cm}}) e^{\pm Y}$$

$$\begin{aligned} \frac{d\sigma}{d\mathcal{T}_B^{\text{cm}}} &= \frac{\sqrt{2}G_F m_H^2}{576\pi E_{\text{cm}}^2} H_{gg}(m_t, m_H^2, \mu) \int dY \\ &\times \int dt_a B_g(t_a, x_a, \mu) \int dt_b B_g(t_b, x_b, \mu) S_B^{gg} \left( \mathcal{T}_B^{\text{cm}} - \frac{e^{-Y}t_a + e^Y t_b}{m_H}, \mu \right) \\ &\times [1 + \mathcal{O}(\Lambda_{\text{QCD}}/m_H, \mathcal{T}_B/m_H)] \end{aligned}$$

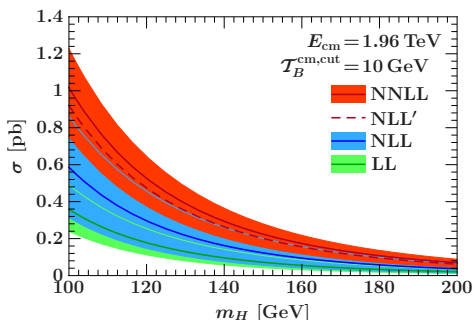
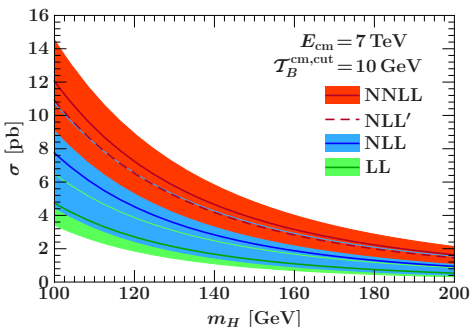


- Gluon hard function contains  $gg \rightarrow H$  vertex plus hard virtual corrections
- Now have gluon beam and soft functions for incoming gluons

# Higgs Production with Central Jet Veto

$gg \rightarrow H$  production cross section with central jet veto  $\mathcal{T}_B^{\text{cm}} \leq \mathcal{T}_B^{\text{cm,cut}}$

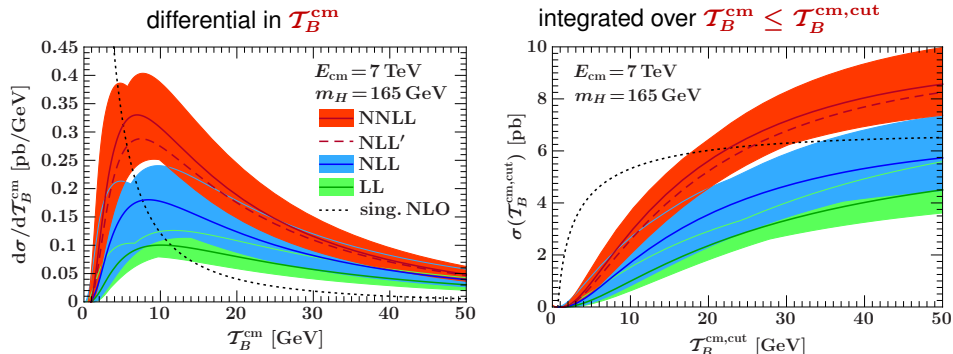
- $\mathcal{T}_B^{\text{cm,cut}} \simeq 10 \text{ GeV}$  gives similar suppression of  $t\bar{t}$  as veto on jets with  $p_T^{\text{jet}} \gtrsim 25 \text{ GeV}$ ,  $|\eta| \lesssim 2.5$  (using Pythia+PGS)



Perturbative corrections and uncertainties significantly larger than in Drell-Yan

- Mostly from fixed-order, NLL' includes NLO matching in NLL result

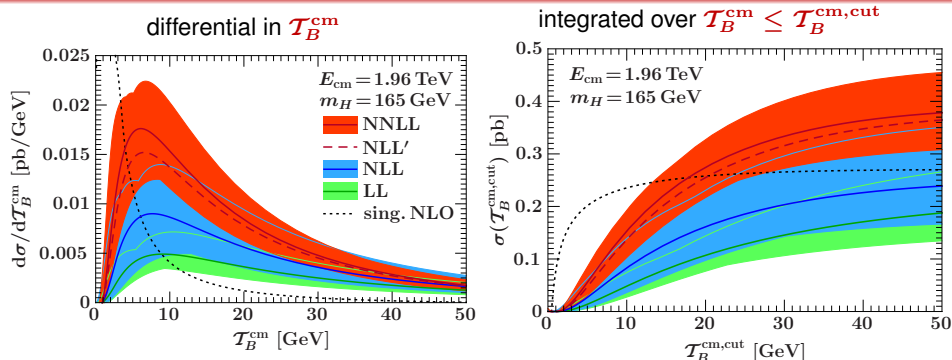
# Beam Thrust Cross Section for Higgs Production



Jet veto has stronger impact for gluons than quarks:  $C_A/C_F = 2.25$

- $\pi^2$  summation enhances, log summation suppresses cross section
- Large  $\sim 25\%$  scale uncertainties even at NNLL
  - ▶ Will be improved by including NNLO matching ( $N^3\text{LL}$ )
  - ▶ NNLO  $H_{gg}(m_t, m_H^2, \mu_H)$  known from three-loop  $ggH$  form factor  
[Harlander *et al.*; Pak, Rogal, Steinhauser]
  - ▶ NNLO  $B_g(t, x, \mu_B)$  and  $S_B^{gg}(k, \mu_S)$  are feasible

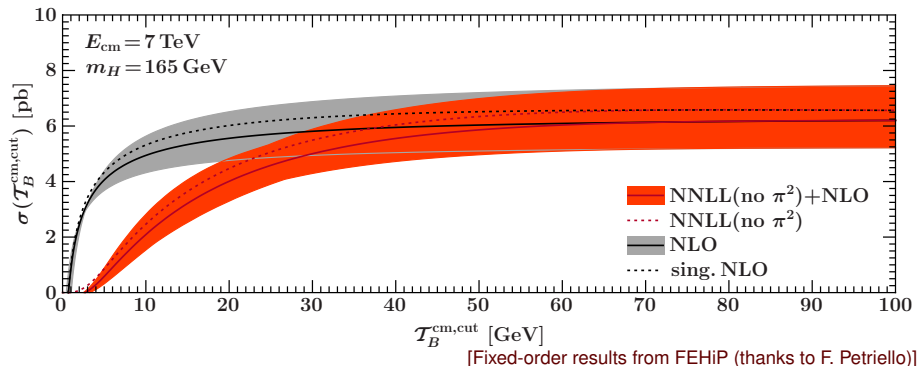
# Beam Thrust Cross Section for Higgs Production



Jet veto has stronger impact for gluons than quarks:  $C_A/C_F = 2.25$

- $\pi^2$  summation enhances, log summation suppresses cross section
- Large  $\sim 25\%$  scale uncertainties even at NNLL
  - ▶ Will be improved by including NNLO matching ( $\text{N}^3\text{LL}$ )
  - ▶ NNLO  $H_{gg}(m_t, m_H^2, \mu_H)$  known from three-loop  $ggH$  form factor  
[Harlander *et al.*; Pak, Rogal, Steinhauser]
  - ▶ NNLO  $B_g(t, x, \mu_B)$  and  $S_B^{gg}(k, \mu_S)$  are feasible

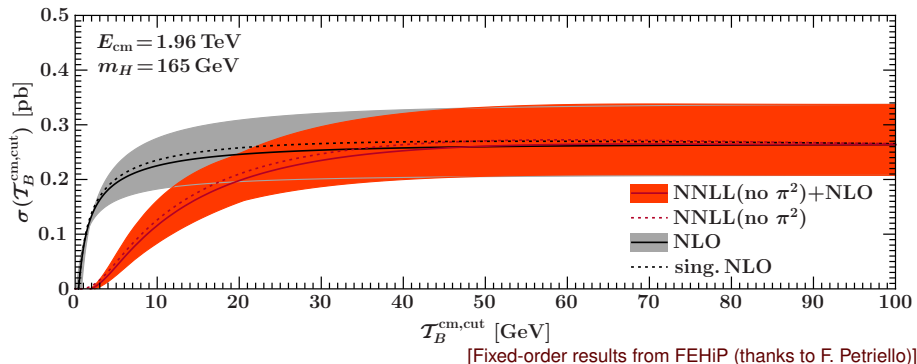
# Matching to Fixed NLO



Match NNLL (without  $\pi^2$ ) to full NLO( $\mu = m_H$ ) at large  $\mathcal{T}_B^{\text{cm,cut}}$

- Nonsingular corrections from full NLO are small
- Need to choose  $\mu_B(\mathcal{T}_B^{\text{cm,cut}})$  and  $\mu_S(\mathcal{T}_B^{\text{cm,cut}})$  with  $\mu_S = \mu_B = \mu_H$  for  $\mathcal{T}_B^{\text{cm,cut}} \gtrsim m_H/2$  to switch off resummation

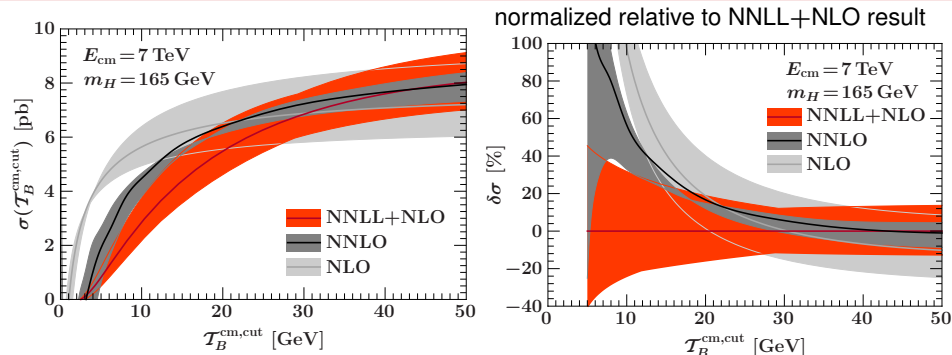
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# Comparison to Fixed NNLO

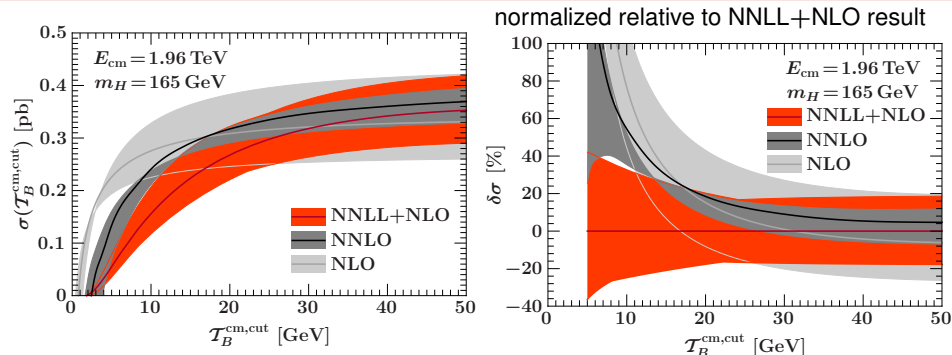


[Fixed-order results from FEHiP (thanks to Frank Petriello)]

Compare full NNLL+NLO to  $\text{NLO}(\mu = m_H/2)$  and  $\text{NNLO}(\mu = m_H/2)$

- NNLO moves NLO into right direction (as it should)
- Total cross section from NNLL+NLO (with  $\pi^2$ ) very close to fixed NNLO
- $\text{N}^3\text{LL} + \text{NNLO}$  required to properly compare to NNLO (will also reduce scale uncertainty)

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# Outline

- 1 Jet Vetos and ISR
- 2 Beam Thrust and Beam Functions
- 3 NNLL Results for Drell-Yan and Higgs
- 4 Outlook

# $N$ -Jettiness for $pp \rightarrow XL + N$ jets

$$\tau_N = \frac{2}{Q^2} \sum_k \min \left\{ \mathbf{q}_a \cdot \mathbf{p}_k, \mathbf{q}_b \cdot \mathbf{p}_k, \mathbf{q}_1 \cdot \mathbf{p}_k, \dots, \mathbf{q}_N \cdot \mathbf{p}_k \right\}$$

Particles get associated with closest jet/beam

- $\mathbf{q}_i$  are massless reference momenta

$$\mathbf{q}_a^\mu = x_a E_{\text{cm}} \frac{1}{2} (1, \vec{z}), \quad \mathbf{q}_J^\mu = E_J (1, \vec{n}_J)$$

- Large contributions only from energetic particles not collinear to any beam or jet

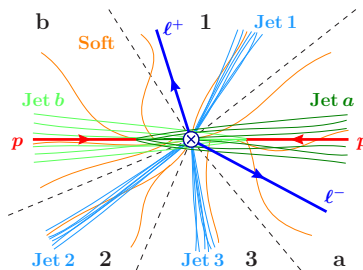
▶  $\tau_N \sim 1$ : Additional hard jets

▶  $\tau_N \ll 1$ :  $N$ -jet region

- Different jet algorithms only differ in treatment of soft particles

▶ (must) give same  $\mathbf{q}_m$  up to p.c.  $\Rightarrow \tau_N^{\text{alg.1}} = \tau_N^{\text{alg.2}} + \mathcal{O}(\tau_N^2)$

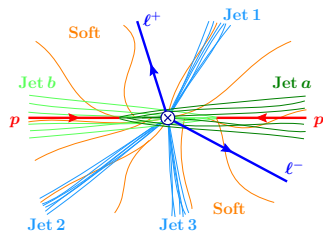
$\Rightarrow$  Implements jet veto via inclusive event shape, can sum logs  $\alpha_s^n \ln^m \tau_N^{\text{cut}}$



# Factorization for $N$ -Jet Cross Section ( $\tau_N \ll 1$ )

$$\begin{aligned}
 \frac{d\sigma^{F_N}}{d\tau_N} &= \int dx_a dx_b \int d^4q d\Phi_L(q) \int d\Phi_N(\{q_J\}) (2\pi)^4 \delta^4\left(q_a + q_b - \sum_J q_J - q\right) \\
 &\times F_N(\{q_m\}, L) \sum_{ij, \kappa} \text{tr} \hat{H}_{ij \rightarrow \kappa}(\{q_m\}, L, \mu) \\
 &\times \int dt_a B_i(t_a, x_a, \mu) \int dt_b B_j(t_b, x_b, \mu) \prod_J \int ds_J J_{\kappa_J}(s_J, \mu) \\
 &\times \hat{S}_N^{ij \rightarrow \kappa}\left(\tau_N - \frac{t_a + t_b + \sum_J s_J}{Q^2}, \{q_m\}, \mu\right)
 \end{aligned}$$

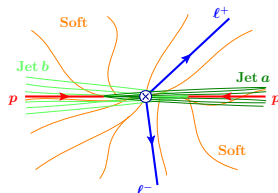
- $\hat{H}_{ij \rightarrow \kappa}(\{q_m\}, L)$  contains hard scattering  
 $i(q_a)j(q_b) \rightarrow L(q)\kappa_1(q_1) \cdots \kappa_N(q_N)$
- Measurement function  $F_N(\{q_m\}, L)$   
 encodes signal requirements on  $L$  and  $N$  jets
- $B_i, B_j$  describe ISR,  $J_{\kappa_J}$  describe FSR
- $\hat{S}_N^{ij \rightarrow \kappa}$  describes soft radiation



# Summary

## Measurements need to veto unwanted jets

- Jet vetos have strong impact and cause large logs
- ⇒ Should be summed beyond parton shower



## Beam thrust: Inclusive event shape to veto jets

- Hadronic observable with full analytic control
- Factorization allows systematic resummation of logarithms
- Can be generalized to processes with  $N$  signal jets:  $N$ -jettiness
- ⇒ Beam functions required to sum double logs from ISR (ISR analog of jet functions)

## Results for two examples with no central jets at NNLL

- Drell-Yan: Important benchmark process to study ISR and jet veto
- Higgs production: Jet veto is essential to beat down  $t\bar{t}$  background

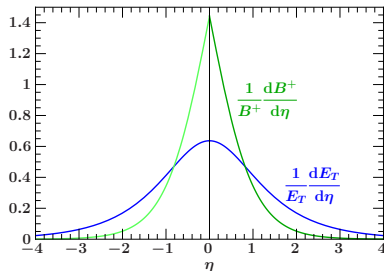
# Backup Slides

# Compare to Transverse Energy and Momentum

$$\tau_B = \sum_k \frac{|\vec{p}_{kT}|}{Q} e^{-|\eta_k - Y|} = \frac{1}{Q} [e^Y B_a^+(Y) + e^{-Y} B_b^+(Y)]$$

$$E_T = \sum_k |\vec{p}_{kT}|, \quad \vec{p}_T = \sum_k \vec{p}_{kT}$$

	$\tau_B$	$E_T$	$\vec{p}_T$
Linear in momenta	✓	✗	✓
Constrains central radiation	✓	(✓)	✗
Insensitive to very forward (detector limit)	✓	(✓)	✓
Boost invariant along z	✓	✓	✓



- $B^+$  is natural scalar quantity complementary to vector  $\vec{p}_T$
- Limited rapidity reach of detectors is no problem, worst-case scenario:

$$\text{LHC:} \quad \eta_{\text{det}} = 5 \Rightarrow 14 \text{ TeV } e^{-10} = 0.6 \text{ GeV}$$

$$\text{Tevatron:} \quad \eta_{\text{det}} = 4 \Rightarrow 2 \text{ TeV } e^{-8} = 0.7 \text{ GeV}$$

# Standard Factorization for Inclusive Drell-Yan

Factorization theorem for inclusive  $pp \rightarrow X \ell^+ \ell^-$  [Collins, Soper, Sterman; Bodwin; '80s]

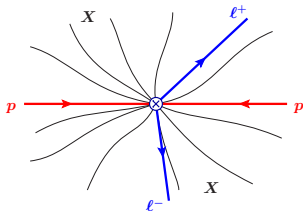
$$\frac{d\sigma}{dQ dY} = \frac{8\pi\alpha_{\text{em}}^2}{9E_{\text{cm}}^2 Q} \sum_{i,j=\{q,\bar{q},g\}} \int \frac{d\xi_a}{\xi_a} \frac{d\xi_b}{\xi_b} \hat{\sigma}_{ij}\left(\frac{x_a}{\xi_a}, \frac{x_b}{\xi_b}, Q^2, \mu\right) f_i(\xi_a, \mu) f_j(\xi_b, \mu) \times \left[1 + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{Q}\right)\right]$$

- $x_{a,b} = \frac{Q}{E_{\text{cm}}} e^{\pm Y}$  leptonic momentum fractions

- ▶ (Only) at tree level:  $\xi_a = x_a, \xi_b = x_b$

- Only physical scales are  $\Lambda_{\text{QCD}}$  and hard scale  $\mu_H \simeq Q$

- ▶ PDF evolution sums single logarithms  $\alpha_s^n \ln^n(\Lambda_{\text{QCD}}/Q)$



⇒ Valid for inclusive Drell-Yan, insufficient to sum double logs from jet veto

# One-Loop Matching Calculation for Beam Function

Compute  $\mathcal{I}_{ij}(t, z)$  perturbatively by taking partonic external states

$$\langle \mathcal{O}_i(t, \omega, \mu) \rangle = \sum_j \int \frac{d\omega'}{\omega'} \mathcal{I}_{ij}\left(t, \frac{\omega}{\omega'}, \mu\right) \langle \mathcal{Q}_j(\omega', \mu) \rangle$$

$$\mathcal{I}_{qq}^{(1)} =$$

$$\mathcal{I}_{gg}^{(1)} =$$

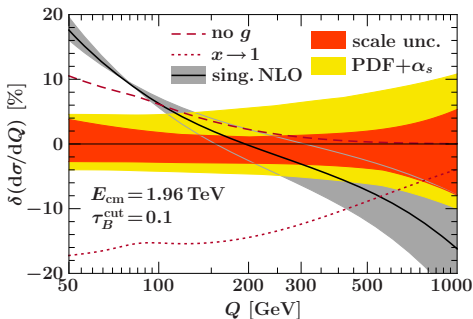
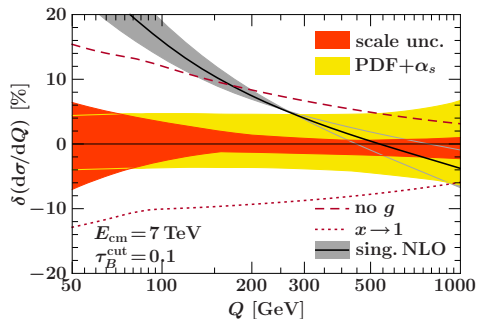
# NLO Result for Quark Beam Function

$$B_i(t, x, \mu) = \sum_j \int_x^1 \frac{d\xi}{\xi} \mathcal{I}_{ij}\left(t, \frac{x}{\xi}, \mu\right) f_j(\xi, \mu)$$

$$\begin{aligned} \mathcal{I}_{qq}(t, z, \mu) &= \delta(t) \delta(1-z) + \frac{\alpha_s(\mu) C_F}{2\pi} \theta(z) \\ &\quad \times \left\{ \frac{2}{\mu^2} \mathcal{L}_1\left(\frac{t}{\mu^2}\right) \delta(1-z) + \frac{1}{\mu^2} \mathcal{L}_0\left(\frac{t}{\mu^2}\right) \left[ P_{qq}(z) - \frac{3}{2} \delta(1-z) \right] \right. \\ &\quad \left. + \delta(t) \left[ \mathcal{L}_1(1-z)(1+z^2) - \frac{\pi^2}{6} \delta(1-z) + \theta(1-z) \left( 1-z - \frac{1+z^2}{1-z} \ln z \right) \right] \right\} \\ \mathcal{I}_{qg}(t, z, \mu) &= \frac{\alpha_s(\mu)}{4\pi} \theta(z) \left\{ \frac{1}{\mu^2} \mathcal{L}_0\left(\frac{t}{\mu^2}\right) P_{qg}(z) + \delta(t) \left[ P_{qg}(z) \left( \ln \frac{1-z}{z} - 1 \right) + \theta(1-z) \right] \right\} \end{aligned}$$

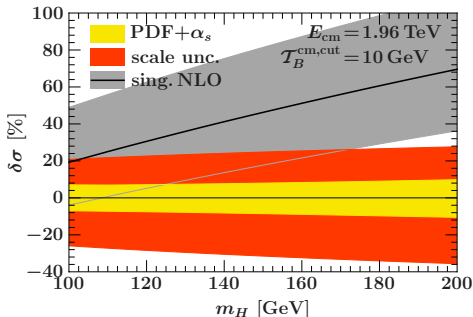
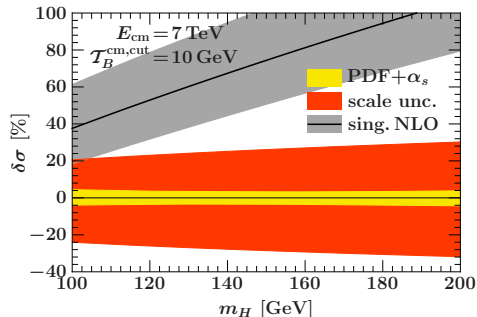
- $P_{qq}(z), P_{qg}(z)$  AP splitting functions,  $\mathcal{L}_n(y) = [\theta(y)(\ln y)^n/y]_+$
- Logs are minimized for  $\mu^2 \simeq t$
- Nontrivial check:  $\mu$  dependence of  $\mathcal{I}_{ij}(t, z, \mu)$  indeed converts PDF running into beam-function running

# Drell-Yan Theory Uncertainties



- Difference between fixed-order and resummed result is generically large and not captured by fixed-order scale uncertainties
- PDF+ $\alpha_s$  uncertainties are MSTW2008 **90%** CL
- Gluon contribution more important at LHC than Tevatron ( $pp$  vs.  $p\bar{p}$ )
- Threshold limit  $x \rightarrow 1$  for beam functions is poor approximation

# Higgs Production Theory Uncertainties



- Difference between fixed-order and resummed result is larger than for Drell-Yan
- Also larger scale uncertainties, **20%** to **30%** at NNLL
- PDF+ $\alpha_s$  uncertainties are MSTW2008 **90%** CL